ISSN 2350-1030

International Journal of Recent Research in Physics and Chemical Sciences (IJRRPCS)
Vol. 8, Issue 2, pp: (13-25), Month: October 2021 – March 2022, Available at: www.paperpublications.org

LEARNER DIFFICULTIES IN SOLVING WORD AND GRAPHICAL PROBLEMS IN QUADRATIC EQUATIONS

[1]G. K., Tonui*, [2]L. Ayiro, [3]K. Ongeti

*Corresponding Author: Dr. Gilbert Kipkoech Tonui, Kabarak University P.O Private Bag 20157, Nakuru, Kenya.
kipkoechtonui@kabarak.ac.ke. Tel. +254-0726928386

Abstract: **Mathematics is the cornerstone of development of any contemporary society since attaining self-reliance requires creative and problem solving individuals who can identify opportunities in their environment. However, a study on learner's difficulties in solving quadratic equations would provide a useful framework for improving teaching and learning in secondary school education. The study aimed to establish those difficulties learners' faces. Vygotzy theory of concept formation guided the study. The study was conducted in Kericho County, Kenya with 384 learners participating in the study. A descriptive survey research design was employed. Learner's diagnostic test instrument containing quadratic equations was designed based on Bloom's cognitive domains of learning was administered. Quantitative data collected was analyzed using both descriptive and inferential statistics based on the objectives of the study. The study found the following difficulties: learners confused intercepts and coordinates, finding equation of a line, wrong table values, did not know what roots of a function means, incorrect graph, gave coordinates and using roots to find equation. Considering these difficulties, teachers should put a lot of emphasize on them during teaching.**

Keywords: **Difficulties, Solving Word and Graphical Problems and Quadratic Equations.**

1. INTRODUCTION

Moreover, quadratic equations and functions in secondary school level usually starts in form two and end at form four. This gives less opportunity for learners to reason, make sense and build up a bridge in relating threshold concepts with solving problems. Hence, results in a gap between high- and low- achiever learners (Susac, Bubic, Vrbanc, & Planinic, 2014). Quadratic equations and functions emphasizes on applications of Mathematics to real life experiences and practical approaches to teaching and learning in an effort to address such contemporary issues as information technology, health, gender and integrity. Learners' Mathematics performance worldwide and Kenya in particular has been unsatisfactory. Mathematics examination results at the Kenya Certificate of Secondary Education (KCSE) for instance have been consistently dismal over the last decade. Findings of most studies globally and in Kenya attribute this unfavorable state of affairs to learner's gender, attitude, school type, teaching strategy among others.

1.1 Statement of the Problem

In contrast to the prominence of secondary school quadratic equations and functions, learners are reported to lack comprehension on threshold concepts. The Ministry of education, (MOE, 2015), while releasing the results of 2014 candidates, reported that the number of candidates with grade A went down from 3,073 in 2014 to 2,636 in 2015 and only 141 for 2016 candidates. However, Mathematics was among the subjects in which candidates performed poorly. The candidates who scored D+ and below were 40% (209,807 out of a total 525,802) in 2015, but in 2016 performance D plain and below were 60% (346,252 out of 577,000 candidates) which implies that the candidates scored less than 34% on average in the seven subjects, Mathematics included. Jupri, Drijvers and van den Heuvel-Panhuizen, (2014) noted that lack of learner comprehension and performance in quadratic equations and functions are related to how it is being taught the threshold concepts in secondary school. In this case, the common way of teaching often leads to learners' comprehension of variables, the arithmetic processes and the strategies to solve the algebraic problems.

1.2 The Purpose of the Study

The purpose of this study was to establish the learner's difficulties in solving word and graphical problems in Quadratic Equations and Functions in Kericho County public secondary schools.

1.3 Objectives

i. To analyze learner difficulties in solving word problems in Quadratic Equations and Functions.

ii. To analyze learner difficulties in solving graphical problems in Quadratic Equations and Functions.

Piaget (1957) proposed that learners develop knowledge by inventing or constructing reality out of their own ideas about how the world works. Piaget viewed intelligence as the individual's way of adapting to new information about the world. They do this through a process of equilibration, which means balancing assimilation (fitting reality into their existing knowledge) and accommodation (modifying schemas to fit reality).

This study targeted form four learners, who according to Piaget (1957) are in the final stage of cognitive development. Consequently, the learners should possess the qualities and characteristics of this stage by having a good comprehension on areas of difficulties as this reflects high order reasoning. Therefore, mental skills running through the six taxonomic levels are tested this study alongside the Piagetian theory so that the intellectual maturity of the learner is nurtured. Therefore, much of the examination questions should test the six levels; more emphasis ought to be placed on the application, analysis, synthesis and evaluation. The diagnostic test questionnaire contains questions constructed with the guide of Bloom's and Piaget's theories. This enabled the study to make judgments on the learners' difficulties in solving word and graphical problems in quadratic equations and functions.

2. LITERATURE REVIEW

2.1 Quadratic Equations and Functions with one Known

Mu'awiya, (2013), while conducting a study on analysis of problem-solving difficulties with quadratic equations among senior secondary schools students in Zaria, Nigeria, used the Jackson-Ashmore model. A total of 126 Senior Secondary 2 Mathematics learners randomly selected from three private schools in Zaria with a mean age of 17 constituted the sample size for the study. The Mathematics achievement tests (MAT), Mathematics competence test (MCT) and Problem-Solving Test in Quadratic equation (PSTQ) were used for the study. The learners were classified as high achievers and low achievers using the categorization test score designed by the investigator. Data were analyzed using facility values (FV), mean, simple percentages (%) and chi-square statistics. The findings from the study showed that learners performed poorly in Mathematical problems involving quadratic equations.

Nielsen, (2015), while conducting a study entitled "understanding quadratic functions and solving quadratic equations: An analysis of learner's thinking and reasoning" sought to learn what high school learners who have completed an Algebra 2 or Pre-calculus class understand about quadratics. This qualitative study employed cognitive interviews of 27 learners in grades nine through eleven. This study took place in a high school in the northwestern United States. Learners in this high school take a series of mathematics courses starting with Algebra 1 in 7th, 8th or 9th grade and then continuing with Geometry, Algebra 2, and possibly Pre-calculus and Calculus. Most learners at this school complete at least Algebra 2. The selection of classrooms and schools was a purposeful convenience sample. Moreover, several Mathematics teachers in the circuit where the study was conducted complained that learners were not performing well in this topic during examinations. Very little is known about learners' understanding of the behavior of quadratics and how the graphs and equations of quadratic functions are related.

2.2 Solving Word Problem

Didiş et al., (2011) found that only one learner, who was also successful in solving all the symbolic equation, was successful in solving all the questions in the word-problems context. While the learners were mostly successful in solving word problem 2(46.1%), they were least successful in solving the word problem 4(3.7%). On the other hand, the

ISSN 2350-1030
International Journal of Recent Research in Physics and Chemical Sciences (IJRRPCS)
Vol. 8, Issue 2, pp: (13-25), Month: October 2021 – March 2022, Available at: www.paperpublications.org

learners who provided the correct answer for the word problems did not necessarily solve the problem using quadratic equations. In particular, while some learners solved the first problem with a guess-and-test strategy, some learners attempted to solve the third problem by initially making a drawing and then examining it from a different point of view, without formulating a quadratic equation to represent the relationship. Learners attempted to use guess-and-test strategy in order to find the number of days that they initially planned to get the order ready.

In other problems, the learners initially drew a triangle to show the data and to see what was going on. Then, these learners recognized that the triangle was familiar with the 3:4:5 triangles, and concluded that the dimensions of the right triangle in the diagram must be a 30-40-50 triangle. Analysis of students' incorrect solutions and the interview data revealed that the reasons for low performance in forming quadratic equations stated as word problems are threefold: learners did not fully comprehend the problem, learners understood the problem, however, they did not know how to represent the information as a quadratic equation (or approach the problem differently), and learners understood the problem and represented the information as a quadratic equation, however, they had difficulty solving the problem and thus, also with interpreting it.

A study by Egodawatte, (2011) on secondary school learners' misconceptions in algebra used a mixed method research design and reported several types of errors and they were categorized into six major groups: Reversal error; most common error was the reversal error: (48%). The majority of learners (84%) used the equal sign to denote equality without considering the proportional relationship of the variables, used the letters as labels instead of a varying quantity. majority learners considered symbols as labels and formed the equation by mapping the sequence of words directly into the corresponding sequence of literal symbols. Guessing without reasoning errors resulted when students apparently solved a problem by guessing-that is, when there was no overt evidence that the stated information was the result of a mathematical operation, performed a mental operation; hence, unsubstantiated outcomes rather than guessing and not able to verify the realness of their answers by use of meta-cognitive abilities such as verification or looking back.

Forming additive or multiplicative totals from proportional relationships, learners attempt to connect the variables in an equation as an additive total. Learners can understand the problem statement; however, they do not know how to represent the given information as a quadratic equation Didiş et al., (2011). Thus, instead of applying Pythagorean Theorem and getting a quadratic equation that represents the relationship among the distances in the situation, the learner set up an incorrect relationship $2x + (2x + 10) = 50$, as if the sum of the distances traveled after two hours was 50 km. It is safe to say that the problem can be solved easily $2x = 30 \ or \ 2(x + 5) = 40$, upon recognizing the triangle as $30 - 40 - 50$. However, the learner failed to apply the Pythagorean Theorem to set up the correct linear relationships. Similarly, in other word problem, some learners did not formulate the correct algebraic relationship between the side length of the square and the width of the rectangle. Although they did comprehend the problem statement, they could not interpret the information presented, in order to form the quadratic equation, or could not set up the quadratic equation in the correct form.

Threshold concepts in grasping the relationship between two or three varying quantities, learners are expected to understand the relationships among the variables, form equation(s), and solve them. Many of the answers indicated that learners used arithmetic methods, working backwards, or guessing to find solutions rather than algebraic methods. Only $5\% \ and \ 14\%$ of the learners used algebraic methods to solve the problems. Didiş et al., (2011) also found that although some learners set up the quadratic equation correctly in the word problems, they made mistakes while solving it. The most common error for the second word problem was that the learners constructed the algebraic relationships and formulated the quadratic equation correctly as; $2a^2 + 6a - 176 = 0$ however, they made calculation errors while using the cross-multiplication method and zero product property. On the other hand, some learners set up the quadratic equation correctly but they could not solve the equation and as such, their solutions were incomplete.

Incorrect reasoning in word problems with a familiar context, learners have to think beyond the given data by constructing an equation from the given data and prove that the rule does not always work by explaining the relation between variables. (14%) of the learners used only the given data to arrive at incomplete or wrong conclusions and another 14% extended their thinking beyond the given data but they could not grasp the relationship between the two variables at the same time. It was difficult for them to understand the changing relationship between two variables and they lack proportional or relational reasoning.

Didiş et al., (2011) while analyzing comprehension of the word problems; the learners' solutions revealed that, since a large proportion of learners either did not comprehend or miscomprehended the text in the word problems, they could not formulate the related equation. Although the length of the paperboard is twice its width, before cutting and making it into an open box (i.e., a rectangular prism), the student symbolizes the dimensions of the open box as x for the width and $2x$ for the length, and then forms the equation by way of a volume formula for a rectangular prism. Here the learners correctly symbolizes the relationships and formulates the quadratic equation, however, his misinterpretation leads to an incorrect solution. The interview data supported the factor that forming quadratic equations were quite challenging for students due to their difficulty comprehending the problem statement. During the interviews in the current study, learners will be asked to express why they did not comprehend the problem statement, nor the information presented within. Miscellaneous forms of incorrect answers given, was significant (22%) showing that the learners' tendency to misinterpret the operation as a multiplication when it is actually a division.

Although learner performance regarding solving quadratic equations stated in symbolic equation was not high, their performance depended on the structural properties of the symbolic form of the quadratic equation, Didiş et al., (2011). Their performance also depended on how effectively they used factorization, completing to the square, and quadratic formula for solving quadratic equations. Although the quadratic equation in a question is factorable, have integer coefficients and its roots are all rational numbers, the quadratic equation in some questions is not factorable; its second coefficient is a rational number, and its roots are irrational numbers.

Data suggest that learners displayed high performance in solving the quadratic equation when it is easy to factor, has rational roots, and students have had more practice and greater procedural abilities in solving these types of equations. However, when it came to the non-factorable structure of the quadratic equation, students are unsuccessful, due to their limited procedural algebraic and arithmetic abilities. This question required more algebraic symbol manipulations and arithmetic operations, with rational and radical numbers, while applying either a quadratic formula or complete square.

2.3 Graphical Method

The graph of quadratic equations and functions looks a little like a cup, and the bottom of the cup is called the vertex. The mouth of the cup keeps getting larger to infinity. For the most part, the region around the vertex is of interested. The cup is upright (vertex down) when $a > 0$, upside down (vertex up) when $a < 0$. There are three important cases of quadratics depending on where the graph crosses the x-axis (these points are called roots or zeros of the equation). In case I, two distinct, real roots, the vertex lies on the opposite side of the x-axis from the rest of the graph and so the curve must cross the x-axis exactly twice. One can see exactly where the roots are from the graph, and they are clearly real numbers. Analytically, this case corresponds to the portion of the quadratic formula under the discriminant being strictly positive:

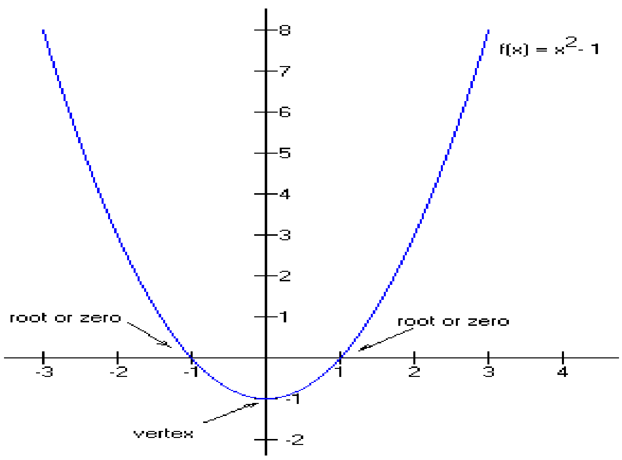

Figure 1: Parabola

A quadratic function is graphically represented by a parabola with vertex located at the origin, below the x-axis, or above the x-axis. Therefore, a quadratic function may have one, two, or zero roots. This method can be used to derive the quadratic formula, which is used to solve quadratic equations. In fact, the roots of the function, $f(x) = ax^2 + bx + c$ are given by the quadratic formula. The roots of a function are the x-intercepts. By definition, the y-coordinate of points

ISSN 2350-1030

International Journal of Recent Research in Physics and Chemical Sciences (IJRRPCS)
Vol. 8, Issue 2, pp: (13-25), Month: October 2021 – March 2022, Available at: www.paperpublications.org

lying on the x-axis is zero. Therefore, to find the roots of a quadratic function, we set $f(x) = 0$, and solve the equation $ax^2 + bx + c = 0$. The vertex is an important coordinate to find because we know that the graph of the parabola is symmetric with respect to the vertical line passing through the vertex. The coordinate of the vertex of a quadratic equation in standard form ($y = ax^2 + bx + c$) is $(-\frac{b}{2a}, f(-\frac{b}{2a}))$, where $x = -\frac{b}{2a}$ and $y = f(-\frac{b}{2a})$.This means that to find the x-value of the vertex in the equation, $y = -3x^2 + x + 1$ use the formula that $x = -\frac{b}{2a}$. In this equation, "b" is the coefficient of the x-term and "a", like always, is the coefficient of the x^2 term (Mathematics Curriculum, 2014).

The study findings by Amidu, n.d., showed that 3 items were presented and analyzed in the content category of ‘recognition of quadratic function for given graphs by using their roots and intercepts’. The average percentage of correct responses for the three items was 35.8%. This shows that the performance of the pre-service teachers in this category was below the 50% average mark and was not very encouraging. Recognizing a quadratic graph with negative co-efficient of and having two distinct roots was categorized by the researcher as the easiest of the three items but only 42.5% ($n = 17$) of the pre-service mathematics teachers were able to supply correct responses. The participants’ worse performance was on ‘recognizing quadratic graph with positive co-efficient of and having two distinct roots’ which only 27.5% ($n = 11$) of the pre-service mathematics teachers was able to supply correct responses.

The results show that pre-service Mathematics teachers were not comfortable with the items in this content category and thus necessitate the study to find out if secondary school learners in Kericho County were able to comprehend threshold concepts in solve quadratic equation and functions with positive coefficient and having 2 distinct roots. Furthermore, the mean percentage score of the participants making the correct match was 39.2%. The question, which required pre-service Mathematics teachers to recognize a new quadratic graph, when the value of the co-efficient of is tripled, had the highest correct responses score (65%, n=26). However, only 27.5% (n=11) and 25.0% (n=10) of the participants were able to recognize the new quadratic graphs when the values of the co-efficient of was halved and became negative. That implies that a few of the respondents were able to get the clue that when the co-efficient of in a quadratic function becomes negative the graph opens downwards.

Parent, (2015) while studying learners’ understanding of quadratic functions which was mainly an investigation of the effects that traditional and multiple representation tasks have on how students think about the quadratic function, specifically the axis of symmetry, vertex, the location of roots, whether the parabola opens up or down, the maximum/minimum point, the y-intercept and the main translations of the function itself when graphed. The specific methods of factoring for roots were not a part of this study. Utilizing a “think-aloud” protocol, each pair participated in the same four tasks. The tasks varied, with one being more traditionally worded, one focused on more multiple representations, and then a combination of the two for two mixed methods tasks. Learners participated in the study tasks over a four-day period (before or after school) for a maximum duration of 45 minutes each day. All six study participants were enrolled in the high school (grades 9-12) where the researcher taught that is located in northern Vermont with a population of approximately 1150 learners. There were four males and two females in the study, two of the males and one female were sophomores (10) and the other three were juniors (11) in high school, making an even split.

While analyzing the data, Parent, (2015) noted the following strategic and misconception observations to be key: Participants preferred the standard form over the vertex form, participants confused the y-intercept of the standard form versus the y-coordinate of the vertex when the function was in vertex form, participants preferred algebraically solving a problem versus tabular or graphical, the linear function term of “slope” came up when learners were discussing the transformations of the quadratic graph, and the learners interpreted the maximum/minimum point of the quadratic function to be the entire (x, y) point of the vertex instead of solely the y-coordinate of the vertex. The data from this study reveals that the participants were limited in both their conceptual and procedural understanding of the quadratic function. The participants illustrated a variety of misconceptions when presented with standard problems related to the quadratic function. But, when given hints through graphs, a function, a formula etc., they were more successful in solving the problem.

On the occasion that learners did understand the function presented to them, they may not have a complete understanding of all of the elements or be able to transfer the function between different representations of it – ordered pairs, table, equation, graph, etc. If a learner only understands a particular form of function, due to that being the only one used in a

ISSN 2350-1030

International Journal of Recent Research in Physics and Chemical Sciences (IJRRPCS)
Vol. 8, Issue 2, pp: (13-25), Month: October 2021 – March 2022, Available at: www.paperpublications.org

course, that a learner will only retain that particular form. Procedural knowledge can allow a learner to pass a class, but conceptual knowledge combined with the procedural knowledge will allow the learner to be prepared for the next mathematical level. As noted by the researchers learners preferred to convert the vertex form of the quadratic to the standard form in order to solve the problem. This is primarily due to the fact that learners see the standard form of most functions more than any other form while taking quadratic functions.

3. RESEARCH DESIGN AND METHODOLOGY

The study targeted all the form four learners in Kericho County. According to county Director of education, there were 152 secondary schools of which 140 were public schools and the rest were private. There were 10,466 form four learners in public secondary schools but the study sampled 384 form four learners. Hence, the county had 24 boys' schools, 23 girls' school and 93 mixed schools. Participating schools were proportionately distributed depending on the number of schools in each stratum.

Participating schools therefore, were randomly selected based on the registration list in Kericho County Education office. Each school was assigned a number corresponding to its serial number in the registration list, folded and shuffled in a basket, and then participating school in each stratum was picked without replacement. The procedure was repeated until the required number of schools in each stratum was attained; 5 boys, 5 girls and 20 mixed schools participated in the study.

4. RESEARCH FINDINGS

4.1 Difficulties in Solving Word Problems

The learners' performance in solving quadratic word problems resulted from difficulties in understanding, representing and translating word problem. In questions 5 and 6, learners were expected to understand the relationships among the variables, form equation(s) and solve them. The performance indicated that learners used arithmetic methods, working backwards or guessing to find solutions rather than quadratic methods. Unlike symbolic questions, most learners had no simple ways to use arithmetic methods in this question other than guessing or using trial and error.

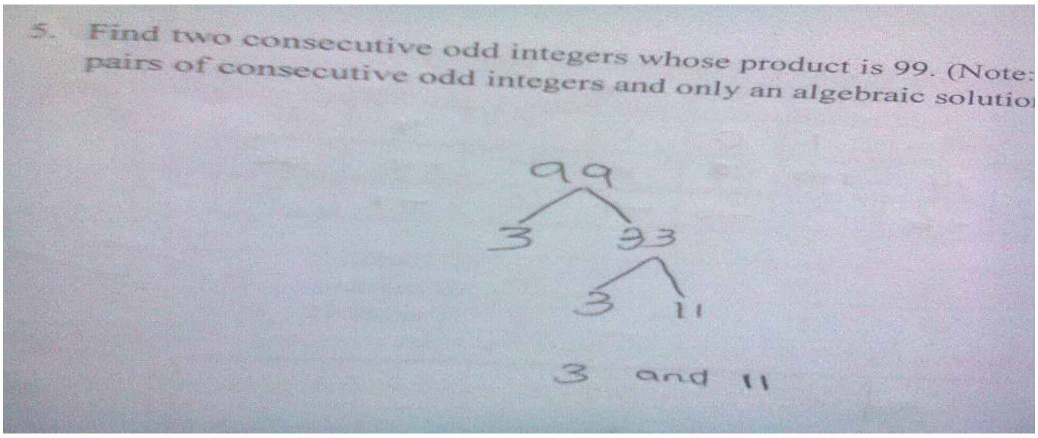

Figure 2: Difficulties of symbolic representation of word problem from Learner 256

In Q5 of the diagnostic test, learners were asked to, "Find two consecutive odd integers whose product is 99"; learners would take the square root to get two numbers, a positive and a negative. The quadratic expression could have looked like, $n(n-2)=99$, to get $n^2+2n-99=0$. So using factorization method to solve, $n^2+11n-9n-99=0$giving$(n+11)(n-9)=0$, then either $(n+11)=0\ or\ (n-9)=0$ and the answer is $n=-11\ or\ n=9$. But the learner instead prime factorize the product just to satisfied the condition of a quadratic solution by just getting two answers of the problem, probably thinking the learner got the question correctly as shown in figure 2 above. This implied that the difficulties might have led to learners' inability to solve the question. Solving word problems involves a triple process: assigning variables, noting constants, and representing relationships among variables. Among these processes, relational aspects of the word problem are particularly difficult to translate into symbols. Thus, learners' difficulties in translating from natural language to quadratic and vice versa are one of the three situations that generally arise when

International Journal of Recent Research in Physics and Chemical Sciences (IJRRPCS)
Vol. 8, Issue 2, pp: (13-25), Month: October 2021 – March 2022, Available at: www.paperpublications.org

learners are in secondary education (Mayer, 1982; Bishop, Filloy & Puig, 2008 as cited in Egodawatte, 2011). Hinsley et al. (1977, as cited in Egodawatte, 2011) showed that the translation of quadratic word problems is guided by schemas. These schemas are mental representations of the similarities among categories of problems. Translation errors frequently occur during the processing of relational statements.

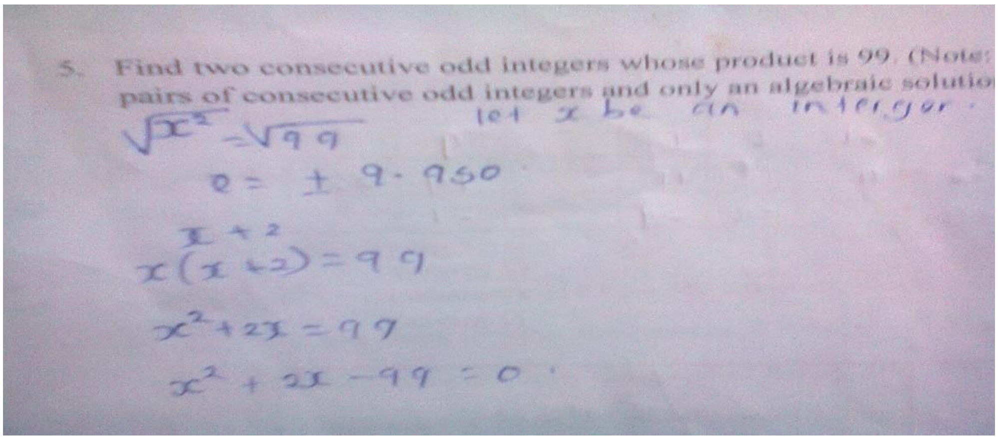

Figure 3: Excerpt from Learner 279 of Inability to Solve Quadratic Equation

Learner 279 had translated the word problem well but the solving difficulty occurred when the student got confused trying to formulate a solution for the problem. Learner 279 set up the quadratic equation correctly as $n^2 + 2n - 99 = 0$, but could not solve the equation using any of the methods and as such, and therefore the solution was incomplete. One noticeable feature in the answers was that 32.6%, 22.8% and 44.6% of learners from boys, girls and mixed schools respectively had difficulties in comprehending the relationship among two varying quantities. In the comprehension phase, a problem-solver comprehends and then forms the text base of the problem, utilizing words as an internal representation in his or her memory. In the solution phase, she or he expresses this internal representation externally and applies the rules of algebra to reach a conclusion (Koedinger & Nathan, 2004; Mayer, 1982 cited in Didiş et al., 2011).

Learners have poor deductive reasoning abilities as 17.5%, 23.1% and 59.4% of them from boys', girls' and mixed schools respectively left this question blank. This gives 59.6% of learners who left the question blank compared to a total of 16.4% of the learners who got the correct solution. Lack of this concepts led to poor performance in quadratic equation. Moreover, Briars and Larkin (1984 in Didis & Erbas, 2015) attributed the word problem-solving difficulty to the learners' psychological processes. They also emphasized factors relating to the problem's features, such as the number of words in the problem, the presence of cue words and the size of the numbers.

Table 1: Difficulties in Solving Word Problem Q6

	Frequency (%)			
Difficulties	**Boys**	**Girls**	**Mixed**	**Total**
Make Drawing and Examining in a Different Point of View	15 (27.3)	15 (27.3)	25 (45.5)	55 (14.3)
Difficulty in Grasping Relation between two Varying Quantities	43 (30.7)	24 (17.1)	73 (52.1)	140 (36.5)
Correct Solution	12 (75.0)	0 (0.0)	4 (25.0)	16 (4.2)
Blank	45 (26.0)	38 (22.0)	90 (52.0)	173 (45.1)
Total	**115 (29.9)**	**77 (20.1)**	**192 (50.0)**	**384 (100)**

The major source of learner's difficulties in solving Q6 was translating the narrative into appropriate quadratic expressions as 27.3%, 27.3% and 45.5% of the learners from boys, girls and mixed schools respectively made drawing and examining it in a different point of view as shown in the figure below.

ISSN 2350-1030

International Journal of Recent Research in Physics and Chemical Sciences (IJRRPCS)
Vol. 8, Issue 2, pp: (13-25), Month: October 2021 – March 2022, Available at: www.paperpublications.org

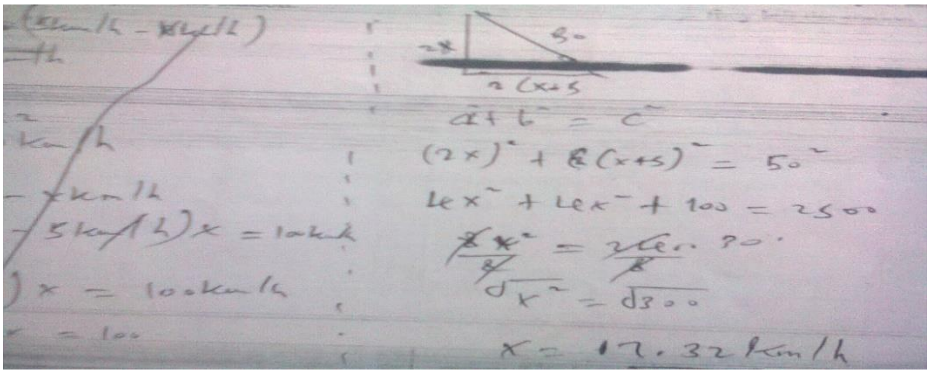

Figure 4: Excerpt of Incorrect Expansion from Learner 30

Learner 30 had a very good understanding of the problem which was expressed with a drawing, but the only difficulty the learner had was the incorrect expansion of $2(x+5)^2$to arrive at $4x^2+100$ instead of $4x^2+20x+10=0$ as shown in the figure below. Building up a single quadratic relationship to satisfy the conditions was so hard for them showing that most students lack the fundamental concept of distinguishing terms used mathematically, Egodawatte, (2011). While citing Clement (1982), Didis, (2015) indicated that learners' lack of comprehension in solving quadratic word problems stem from the difficulties they have in symbolizing meaningful relationships within quadratic equations.

Difficulty in grasping relation between two varying quantities was found with 30.7%, 17.1% and 52.1% of learners from boys, girls and mixed schools respectively and this result from the semantic structure and memory demands of the problem. Consequently, these learners might have translated the syntax of the relational statement into quadratic expression without considering the magnitude of the relationship. Although the linguistic form of the problem's text conveys the significant factors that affect the comprehension process, Stacey and MacGregor (2000 cited in Didis, 2015) claimed that a major reason for lack of comprehension of threshold concept with word problems arises from logic of a problem. They argue that because of their prior experiences with arithmetic word problems, learners perceive the problem-solving process as a series of calculations and shift their thought process from quadratic thinking to arithmetic thinking when solving quadratic word problems.

Operative reasoning is a concept learners used to perform hypothetical operations on two quantities to match the symbols with the words. But 26.0%, 22.0% and 52.0% of learners from boys, girls and mixed schools respectively left the question blank expressing lack of comprehension of threshold concepts of mathematical relationship, Weinberg (2007 as cited in Egodawatte, 2011).

4.2 Difficulties in Solving Graphical Problems

Parent, (2015) described a function as, a relation in which the first coordinate is never repeated. There is only one output for each input, so each element of the domain is mapped to exactly one element in the range.

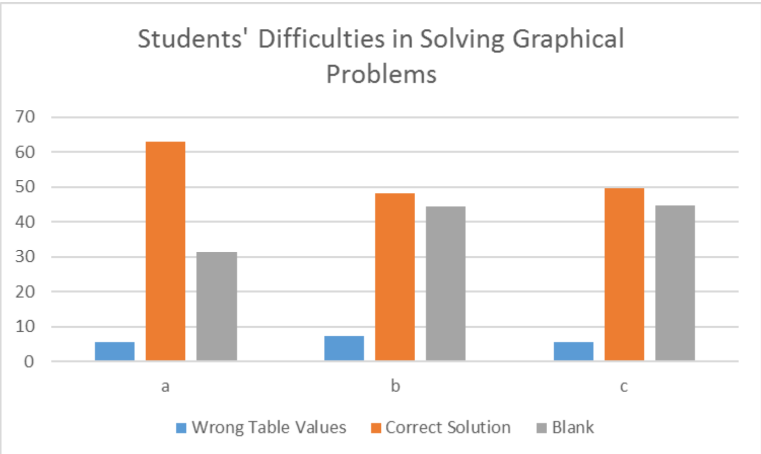

Figure 5: Learners' Performance in Q7

In this study, the difficulties of graphical problems was narrowed down to quadratics and learners were first required to use the given range of values of x to get the corresponding values of y. the following figure shows learners' performance in Q7.

Table 2: Difficulties in Graphical Solution Q.7

	Frequency (%)	
Difficulties	**I**	**II**
Other Transformation	20 (5.2)	15 (3.9)
Confusing Intercepts and Coordinates	7 (1.8)	7 (1.8)
Finds Equation of a Line	51 (13.3)	50 (13)
Correct Solution	12 (3.1)	15 (3.9)
Blank	294 (76.6)	312 (81.3)
Total	**384 (100)**	**384 (100)**

In plotting a graph learners showed a difficulty of not able to find the corresponding values of y within the given range of the values of x as 5.5%, 7.3% and 5.5% of the learners had wrong table values in Q 7 a, b and c respectively. Similarly, Ellis and Grinstead (2008, as cited in Parent, 2015) reported that when working with quadratic functions, learners' difficulties mainly appear with connections between algebraic, tabular, and graphical representations, a view of graphs as whole objects, struggles to correctly interpret the role of parameters, and a tendency to incorrectly generalize from linear functions. Learners with correct solutions question a, b and c were 63%, 48.2% and 49.7% respectively. But some of the learners left the question blank as indicated by 31.5%, 44.5% and 44.8% in question a, b and c respectively.

Table 3: Difficulties in Finding Roots of a Problem Q.8

	Frequency (%)				
Difficulties	**a**	**bi**	**b ii**	**b iii**	**b iv**
Wrong table Values	19(4.9)	0.0(0.0)	0.0(0.0)	0.0(0.0)	0(0.0)
Does not Know roots of a function	0.0(0.0)	36 (9.4)	0.0(0.0)	0.0(0.0)	0(0.0)
Incorrect Graph	0.0(0.0)	0.0(0.0)	9.0(2.3)	0(0.0)	0(0.0)
Gives Coordinates	0.0(0.0)	0.0(0.0)	0.0(0.0)	20(5.2)	0(0.0)
Use Roots find Equation	0.0(0.0)	0.0 (0.0)	0.0(0.0)	0.0(0.0)	8(2.1)
Correct solution	337(87.8)	74(19.3)	222(57.8)	137(35.7	18(4.7

Question 8 was a simultaneous quadratic equation and function and in part a 4.9% of the learners had wrong table values, 87.8% got the question correct and 7.3% left the question blank.

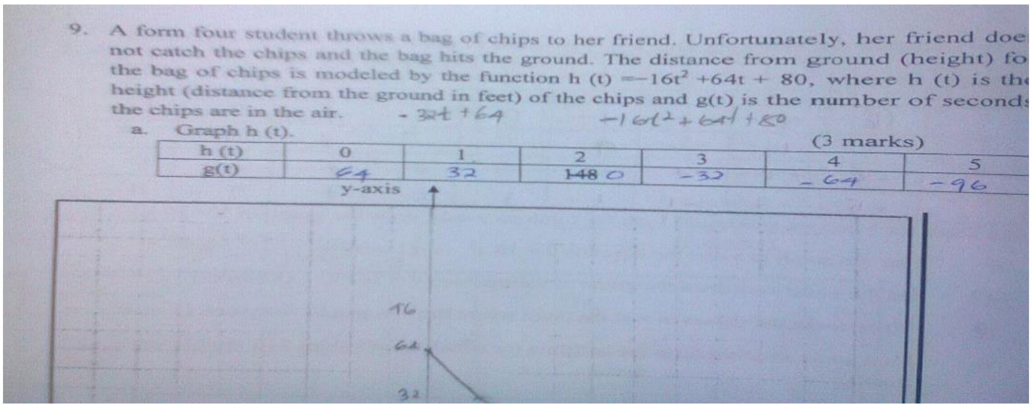

Figure 6: Wrong Table Values for Learner 86

In question 8 the values of Y within the given range of x were $-6, 1, 6, 9, 10, 9, 6, 1$ *and* -6 which the learners were supposed to draw the graph. Question 8 bi, 9.4% of the learners gave the x-intercept of the function as the solution of the equation$0 = 10 - x^2$. There were 19.3% of the learners who got the question correct but majority, 71.4% left the question blank. Question 8 b (ii) required the learners to draw the graph of the equation$y = 2x + 3$; equation of a straight

ISSN 2350-1030

International Journal of Recent Research in Physics and Chemical Sciences (IJRRPCS)
Vol. 8, Issue 2, pp: (13-25), Month: October 2021 – March 2022, Available at: www.paperpublications.org

line, 2.3% of them had incorrect graphs, 57.8% got the question correct while 39% left the question blank. The excerpt of learner 33 below indicates an incorrect graph with a y intercept being 0 instead of 80. The learner used the wrong y values within the range of x coordinates.

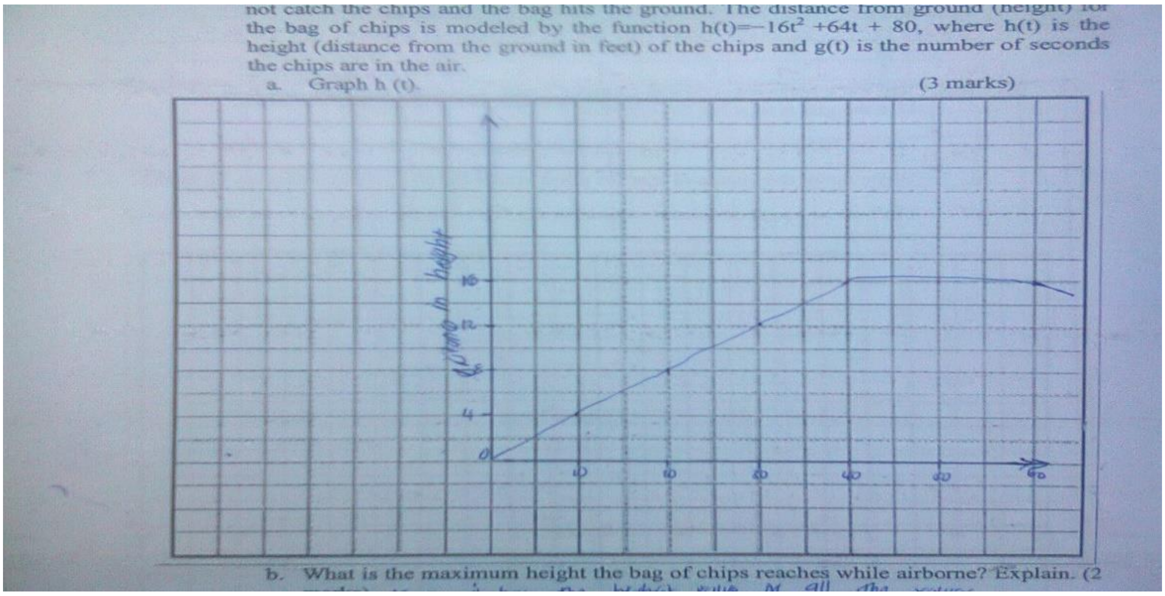

Figure 7: Excerpt of Incorrect Graph of Learner 33

While in question 8 b (iii), learners were required to give the values of x at the point where the two graphs intersected, hence $x = -3.83$ *or* $x = 1.83$. But 5.2% gave the coordinates of the points instead meaning they didn't differentiate between coordinates and values of x. 35.7% of the learners got the correct solution but 59.1% left the question blank. Parent, (2015) reported that when looking at various graphs, though, and indicating the location roots of x from the graphs, learners appeared to not understand the task and what was being asked of them. Instead of simply looking at the quadratic graphs and observing the point of the functions that touched or crossed the x-axis.

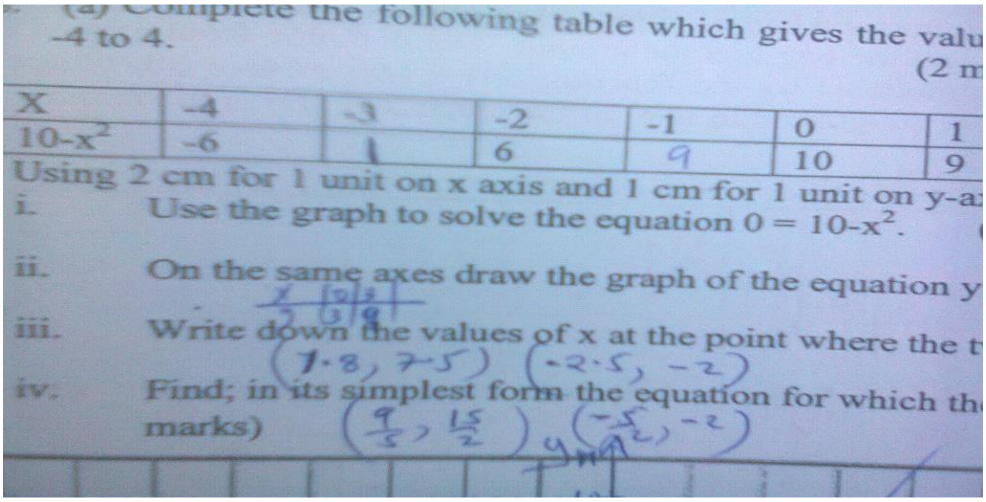

Figure 8: Learner 55 gave Coordinates of the Points of Intersection

In the last question; 8 b (iv) the equation from the roots of x was $x^2 + 2x - 7$. The results showed that majority of the learners didn't know how to form the equation using values of x as roots as 2.1% used roots of x to find linear equation instead of a quadratic equation. The excerpt of learner 55 above gave the coordinates of the point of intersection and was unable to form an equation just using the x-coordinates of this point. A few got the question correct as 4.7% had correct solution while 93.2% didn't attempt the question and left it blank.

ISSN 2350-1030

International Journal of Recent Research in Physics and Chemical Sciences (IJRRPCS)
Vol. 8, Issue 2, pp: (13-25), Month: October 2021 – March 2022, Available at: www.paperpublications.org

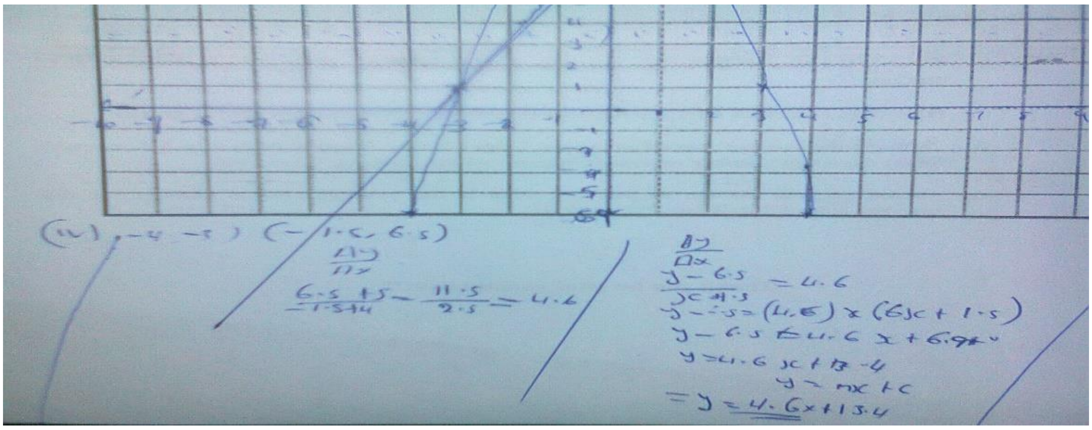

Figure 9: Learner 33 formed Linear Equation

Learner 33 formed linear equation instead of forming a quadratic equation. This shows that the student lack the idea of formation of quadratic equations using the given roots of x.

Table 4: Difficulties in Solving Real Life Problem

	Frequency (%)				
Q9: Difficulties	**a**	**b**	**c**	**d**	**e**
Wrong tables	15.0 (3.9)	0.0 (0.0)	0.0 (0.0)	0.0 (0.0)	0.0 (0.0)
Wrong scale	7.0 (1.8)	0.0 (0.0)	0.0 (0.0)	0.0 (0.0)	0.0 (0.0)
Gives range only	0.0 (0.0)	0.0 (0.0)	0.0 (0.0)	18.0(4.7)	0.0 (0.0)
Gives height of the form fours	0.0 (0.0)	10.0 (2.6)	0.0 (0.0)	0.0 (0.0)	0.0 (0.0)
Gives the max.	0.0 (0.0)	0.0 (0.0)	0.0 (0.0)	0.0 (0.0)	16.0 (4.2)
Correct solution	124 (32.3)	319 (83.1)	335 (87.2)	30 (7.8)	21 (5.5)
Blank	238 (62.0)	55 (14.3)	49 (12.8)	336 (87.5)	347 (90.4)
Total	**384 (100)**	**384 (100)**	**384 (100)**	**384 (100)**	**384 (100)**

The emphasis on functions as a unifying Mathematical concept, as a representation of real-world phenomena, and as an important Mathematical structure was central to this study. Question 9 was a real-life situation where the learners were required to apply the knowledge of quadratic equation and functions to solve. The value of $h\,(t)$ with respect to the corresponding values of $g(t)$ was $80, 128, 144, 128, 80, 0, -112$. In question 9a 3.9% of the learners had incorrect table values, 1.8% used the wrong scale, and 32.3% got the correct solution while 62% left the question blank.

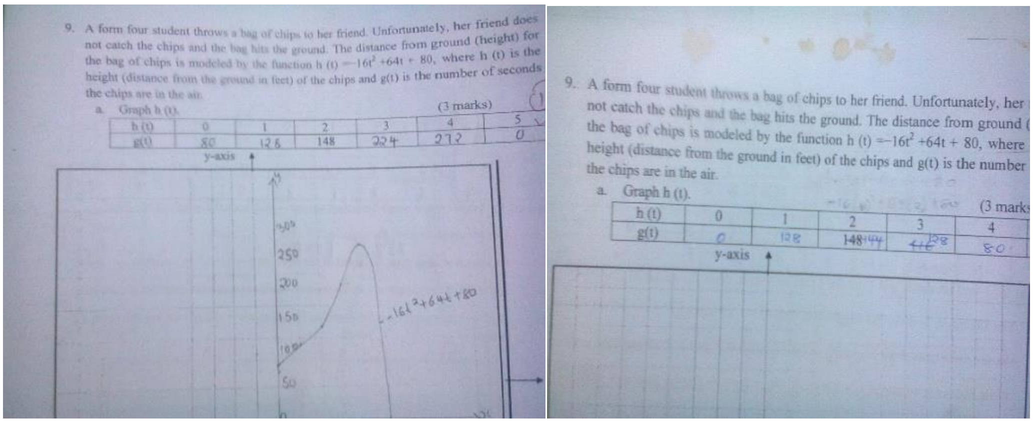

Figure 10: Wrong Graphs for Learners 120 and 125

In question 9b, learners were asked to give the maximum height the bag reaches while airborne which was 144ft, but 2.6% gave the height of the form fours in the bus, 83.1% got the correct solution and only 14.3% left the question blank. The excerpt of learners 120 and 125 in the figure above indicated that the learner had the difficulty of not plotting a good graph due to incorrect y coordinates for the given range of x.

International Journal of Recent Research in Physics and Chemical Sciences (IJRRPCS)
Vol. 8, Issue 2, pp: (13-25), Month: October 2021 – March 2022, Available at: www.paperpublications.org

Majority of the learners, 87.2% were able to find the time taken after the bag was thrown did it hit the ground and only 12.8% left the question blank. Learner 115 in figure 4.33 below differentiated the quadratic equation and solved for t instead of reading it direct from the graph.

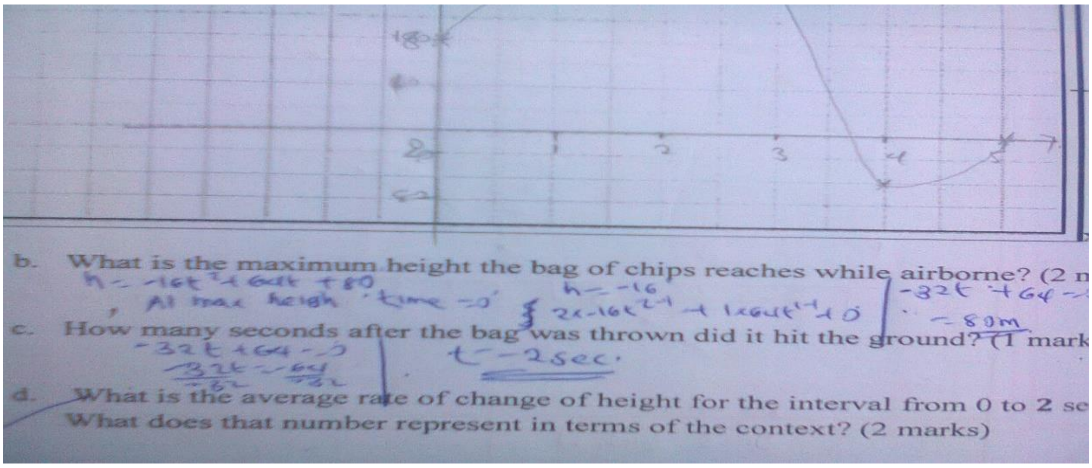

Figure 11: Excerpt of Incorrect Time taken from Learner 115

When learners were asked for the range of change of height for the interval from 0-2 seconds most of them were not able to understand and 4.7% gave the range only without taking the average, 7.8% had correct solution and the rest 87.5% left the question blank. In question 9e, 4.2% of the learners gave the maximum height of the function when they were asked to give the height of the form four learners while 5.5% had correct solution and 90.4% left the question blank. From table 4.12 results, only questions 9b and 9c in which slightly over 300 learners got the correct solution but in questions 9 a, d and e less than half of the learners got the correct solution. This implies that learners cannot comprehend threshold concepts in real life situation problems. This would pose a great challenge to the learners and affect their performance in quadratic equations and functions in the national examinations. The difficulty in comprehension of the concepts was also reported by Ellis and Grinstead (2008, as cited in Parent, 2015) that in quadratic functions, learners' difficulties in tabular, and graphical representations and tend to incorrectly generalize from linear functions.

5. CONCLUSION

Solving word problems involves assigning variables, noting constants, and representing relationships but relational aspects of the word problem are particularly difficult for learners to translate into symbols. Thus, learners' difficulties in translating from natural language to quadratic arise when learners are in secondary education. Hinsley et al. (1977, as cited in Egodawatte, 2011) showed that the translation of quadratic word problems is guided by schemas. These schemas are mental representations of the similarities among categories of problems. Translation errors frequently occur during the processing of relational statements.

In plotting a graph learners showed a difficulty of not able to find the corresponding values of y within the given range of the values of x. learners' difficulties mainly appear with connections between algebraic, tabular, and graphical representations, a view of graphs as whole objects, struggles to correctly interpret the role of parameters, and a tendency to incorrectly generalize from linear functions.

Learners gave the coordinates of the points instead meaning they didn't differentiate between coordinates and values of x. This indicated that learners had difficulty in understanding the task and what was being asked. Learners looked at the quadratic graphs and observing the point of the functions that touched or crossed the x-axis.

6. RECOMMENDATION

From the results, the study can recommend teachers to be more conscious of the knowledge used when learners make mistakes. Teachers should more often emphasize learners' difficulties like the difference between the y-intercept versus the y-coordinate of a vertex, when a function is in any form,

International Journal of Recent Research in Physics and Chemical Sciences (IJRRPCS)
Vol. 8, Issue 2, pp: (13-25), Month: October 2021 – March 2022, Available at: www.paperpublications.org

REFERENCES

[1] Amidu, B. (n.d.). *The use of derive 5 in teaching quadratic functions: a case study at Atebubu College of education*.

[2] Didis, M. G., & Erbas, A. K. (2015). Performance and difficulties of students in formulating and solving quadratic equations with one unknown. *Educational Sciences: Theory and Practice*, *15*(4), 1137–1150.

[3] Didiş, M., Baş, S., & Erbaş, A. (2011). *Students' reasoning in quadratic equations with one unknown*. Presented at the Proceedings of the 7th Congress of the European Society for Research in Mathematics Education. Rzeszów, Poland.

[4] Egodawatte, G. (2011). Secondary school students' misconceptions in algebra. *Department of Curriculum, Teaching, and*.

[5] Hoadley, S., Tickle, L., Wood, L. N., & Kyng, T. (2015). Threshold concepts in finance: conceptualizing the curriculum. *International Journal of Mathematical Education in Science and Technology*, *46*(6), 824-840.

[6] Jupri, A., Drijvers, P., & van den Heuvel-Panhuizen, M. (2014). Difficulties in initial algebra learning in Indonesia. *Mathematics Education Research Journal*, *26*(4), 683–710.

[7] Mu'awiya, A. (2013). An assessment of readiness of the Nigerian building design firms to adopt building information modelling technology. In *The Fifth International Conference for Construction Engineering and Project Management, ICCEPM California*.

[8] Nielsen, L. E. J. (2015). U*nderstanding quadratic functions and solving quadratic equations: an analysis of student thinking and reasoning.*

[9] Parent, J. S. S. (2015). *Students' understanding of quadratic functions: Learning from students' voices*.

[10] Piaget, J. (1957). Logic and psychology.